# Introduction to Quantum Computing & An Application in Power Systems

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Parts of this talk are taken from Carleton Coffrin's talk on the same topic, with permission.
Reading: G. Nannicini, "An Introduction to Quantum Computing, without the Physics", 2020

# What this talk is about? And what not?

Basic Introduction of Quantum Computing without Requiring Physics Knowledge

A Lot of Linear Algebra & Notations!

Not about how to solve your favorite problems using Quantum Computers

Not an expert talk on all things Quantum Computing

# Two Part Talk:
# I. Basics of Quantum Computing
# II. Demystifying Quantum Power Flow

# Part I: Basics of Quantum Computing

# Quantum Computing

**What is Quantum Computing?**

- Computation and information processing based on `quantum mechanics` principles.
- Contrasts with traditional methods reliant on *classical physics*.

**Foundations of Quantum Mechanics**

- Describes nature at its most fundamental level.
- Developed in the early 20th century to explain subatomic particle behavior.

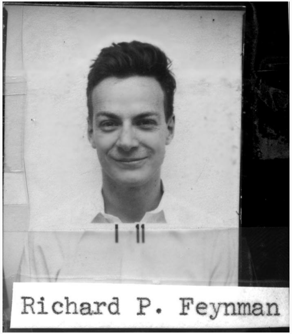
**Success and Impact**

- Underpins fields like Quantum Field Theory and Quantum Chromodynamics.
- Explains microscopic physical phenomena with exceptional accuracy.

# Origin of Quantum Computing: A Vision from Feynman

- **Feynman's Pivotal Question (1981):**

  *"Can probabilistic computers simulate quantum mechanics?"*
  His conclusion: **NO!**

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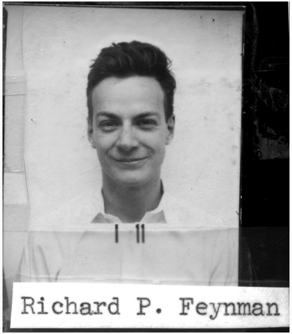


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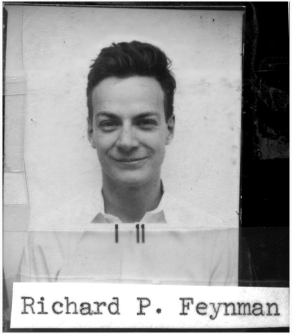
- **The Birth of Quantum Computing:**
  - Inspired by Feynman and contributions from David Deutsch, Paul Benioff, and Yuri Manin.
  - Introduced the concept of computation fundamentally based on *quantum mechanics*.

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Also, There is The Measurement Issue!

# Quantum Computers are Here!

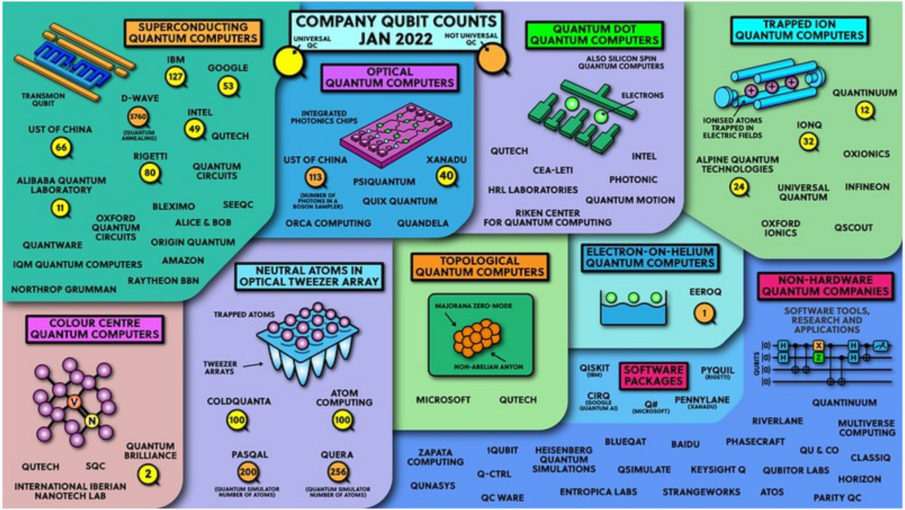



**Figure 2:** Domain of Science - Dominic Walliman

Most important point This introduction is mostly about Notation!

The gate model of quantum computing is just linear algebra over complex numbers with A LOT of special notations!

# Brief Refresher on Complex Numbers

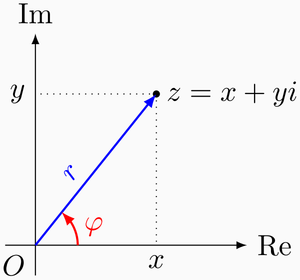
**A complex number has two parts:**

- "Real" part: Along the x-axis
- "Imaginary" part: Along the y-axis

**Can be represented as a magnitude $r$ and an angle $\phi$.**

**Two important operations:**

- Complex Conjugate: $Z^* = x - yi$
- Conjugate Transpose: $A^*$, $A^T$, $A^\dagger$, or $A^H$




$$Z = x + yi = r\angle\varphi$$
$$Z^* = x - yi = r\angle - \varphi$$

Quantum physics is made of waves $\implies$ Has amplitude & angle

$\implies$ Complex number are suitable for representation.

https://en.wikipedia.org/wiki/Complex_number https://en.wikipedia.org/wiki/Conjugate_transpose

# The Kronecker Product $\otimes$ (a.k.a. Tensor Product)

- A recursive matrix operation.
- Efficiently handles exponentially large structures.

$$\mathbf{u} \otimes \mathbf{v} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \otimes \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 v_1 \\ u_1 v_2 \\ u_2 v_1 \\ u_2 v_2 \\ u_3 v_1 \\ u_3 v_2 \end{pmatrix}$$

$$\mathbf{A} \otimes \mathbf{B} = \begin{pmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{pmatrix}$$

**Example:**

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \otimes \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix} & 2 \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix} \\ 3 \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix} & 4 \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 & 5 & 0 & 10 \\ 6 & 7 & 12 & 14 \\ 0 & 15 & 0 & 20 \\ 18 & 21 & 24 & 28 \end{pmatrix}$$

# Practice: Kronecker Product $\otimes$

Given the following matrices:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix},$$

calculate the Kronecker product $\mathbf{A} \otimes \mathbf{B}$.

**Class Question:** Can anyone tell what the first block $(a_{11}\mathbf{B})$ in the top-left corner will look like?

$$\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

- $a_{11} = 1$: $\quad 1 \cdot \mathbf{B} = \begin{pmatrix} 0 & 5 \\ 6 & 7 \end{pmatrix}$

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**Final Answer: Combine into a 4x4 matrix**

$$\mathbf{A} \otimes \mathbf{B} = \begin{pmatrix} 0 & 5 & 0 & 15 \\ 6 & 7 & 18 & 21 \\ 0 & 10 & 0 & 20 \\ 12 & 14 & 24 & 28 \end{pmatrix}.$$

# Dirac Notation (BraKet) $\langle | \rangle$

- **Given a complex Euclidean space** $\mathbb{S} \equiv \mathbb{C}^n$:

  $|\psi\rangle \in \mathbb{S}$ denotes a column vector.

  $\langle\psi| \in \mathbb{S}$ denotes a row vector that is the conjugate transpose of $|\psi\rangle \in \mathbb{S}$:

$$\langle\psi| = |\psi\rangle^\dagger$$

- $\langle\psi|$ is called a **bra**.
- $|\psi\rangle$ is called a **ket**
- A sequence of matrix operations (applied **right to left**) looks like:

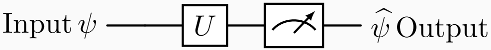
$$\langle\psi_2| \ldots CBA \, |\psi_1\rangle \, .$$

- **What does this have to do with Quantum Computing?**

  Quantum computers work with large complex Euclidean spaces ($\mathbb{C}^n$).

# What is a Gate-Based Quantum Computer?

- The quantum computer has a **state** ($\psi$) that is contained in a quantum register and is initialized in a predefined way.
- The state evolves by applying **operations** ($U$) specified in advance in the form of an algorithm.
- At the end of the computation, some information on the state of the quantum register is obtained by means of a special operation, called a **measurement**.




- There exists an alternative model of computation, called the **adiabatic model**. It is equivalent to the circuit model [Aharonov et al., 2008].

# State of A Quantum Computer

# The Standard Basis of One Qubit

- The Qubit's state is a *unit vector* of 2 complex numbers (i.e., $\mathbb{C}^2$).
- The standard basis is $|0\rangle$ for the first entry and $|1\rangle$ for the second entry.

**Standard Basis**

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

**Unit Vector Requirement**

$$\langle \psi | \psi \rangle = 1$$

**Two Qubit Example**

- The quantum state space grows as $(\mathbb{C}^2)^{\otimes q}$, where $q$ is the number of Qubits.
- For two Qubits, we have:

$$(\mathbb{C}^2)^{\otimes 2} = \mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$$

with the following 4 basis elements: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

`Quantum state vector grows exponentially:` $2^{\#\text{Qubits}}$

# A Trick

**How to write a $0, 1$ string for a Qubit?**

**Example:** $|010\rangle$

- $|010\rangle$ has 3-Qubit $\implies$ $(\mathbb{C}^2)^{\otimes 3} = \mathbb{C}^8$ 8-Dimensional Vector
- If $010$ is in binary, decimal equivalent is $2$
- Place '$1$' at $2^{nd}$ location of 8-Dimensional Vector with index starting from 0

$$|010\rangle = |0\rangle \otimes |1\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Secret Sauce of Quantum: Superposition

- `The quantum system is a linear combination of basis states`

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

- $\alpha, \beta$ are coefficient of basis states & are complex number
- $\alpha$ and $\beta$ represent probability of finding $|0\rangle$ and $|1\rangle$ respectively. Thus

$$\alpha^2 + \beta^2 = 1$$

- Square because these are complex number and sum is one as they are probabilities

**Very Important**

Superposition has `NO` classical equivalent
Bits are always in basis state $\implies$ String of $0s$ and $1s$

# Secret Sauce of Quantum: Entanglement

- A quantum system state that cannot be expressed as the Kronecker product of other states is an **entangled state**.
- Observed values of each Qubit (when measured) are correlated.
- Requires at least **2 Qubits**.

**"Bell Pair"**

(a.k.a. maximally entangled state):

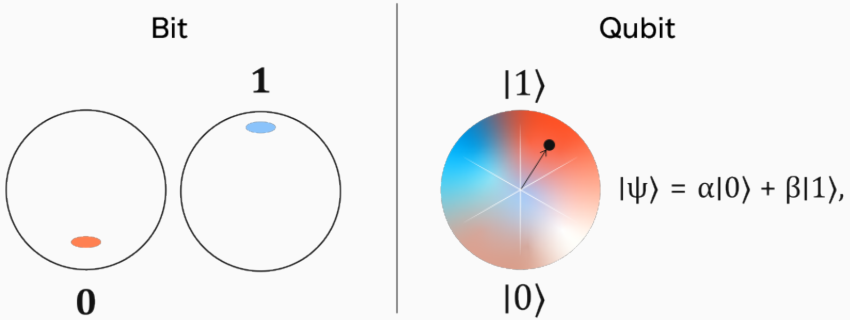
$$|\psi\rangle = \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} \sqrt{0.5} \\ 0.0 \\ 0.0 \\ \sqrt{0.5} \end{pmatrix}, \quad p_s = \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 0.5 \\ 0.0 \\ 0.0 \\ 0.5 \end{pmatrix}$$

There is `No Setting` of constants $c_1$ to $c_8$ such that the Kronecker product produces the Bell state:

$$\begin{pmatrix} c_1 + c_2 i \\ c_3 + c_4 i \end{pmatrix} \otimes \begin{pmatrix} c_5 + c_6 i \\ c_7 + c_8 i \end{pmatrix} \neq \begin{pmatrix} \sqrt{0.5} + 0.0i \\ 0.0 + 0.0i \\ 0.0 + 0.0i \\ \sqrt{0.5} + 0.0i \end{pmatrix}$$

**Note:** If you read "0" on Qubit 1, then you will also read "0" on Qubit 2.

# Bit v/s Qubit




Source: https://www.qnulabs.com/blog/quantum-101-Qubit

# Reading a Quantum State

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- The amplitudes of the Quantum State ($\psi$) determine the probability distribution of the basis states that you will observe.

`Probability Distribution` $p_s = \text{diagonal}(|\psi\rangle\langle\psi|)$

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# Step-by-Step: Calculating $p_s$ from $|\psi\rangle$

$$|\psi\rangle = \begin{matrix} |0\rangle \\ |1\rangle \end{matrix} \begin{pmatrix} 0.5 + 0.25i \\ \frac{\sqrt{2}}{4} + 0.75i \end{pmatrix}.$$

**Step 1: Find the Probability of Each Basis State ($p_s$)**

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**Step 2: Compute the Magnitudes of Each Amplitude**

For $|0\rangle$: $|0.5 + 0.25i|^2 = (0.5)^2 + (0.25)^2 = 0.25 + 0.0625 = 0.3125 = \frac{5}{16}$.

For $|1\rangle$: $|\frac{\sqrt{2}}{4} + 0.75i|^2 = \left(\frac{\sqrt{2}}{4}\right)^2 + (0.75)^2 = \frac{2}{16} + 0.5625 = 0.625 = \frac{11}{16}$.

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The probability distribution $p_s$ is:

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**The probabilities of measuring $|0\rangle$ and $|1\rangle$ are $\frac{5}{16}$ and $\frac{11}{16}$, respectively.**

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**Goal of a Quantum Algorithm:**

- `Put most of the state probability into the "right" basis vector` (e.g., optimal solution to a combinatorial problem).
- `Then reading the solution out of the quantum computer is easy.`

# Quantum Gates

- **Quantum Operations** $(U)$ are called **Gates**.
  - A square matrix of complex numbers applied to the quantum state $(\psi)$.
  - Theory of quantum mechanics requires these operations to be **unitary**.
  - Given the state vector has size $\mathbb{C}^{2^q}$, the gates have size $\mathbb{C}^{2^q \times 2^q}$, where $q$ is the number of Qubits.
- **Unitary Property:**

$$U^\dagger U = UU^\dagger = I$$

- **Important Consequences:**

  Quantum operations are linear.

  Quantum operations are reversible.

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**Note**

While these properties may seem to be extremely restrictive, a quantum computer is **Turing-complete!** [Deutsch1985]

$\Longrightarrow$ It can do whatever a classical computer can do, and more.

# Quantum Gates, One Qubit Example

$$U = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad |\psi_0\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

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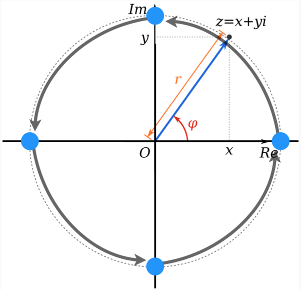
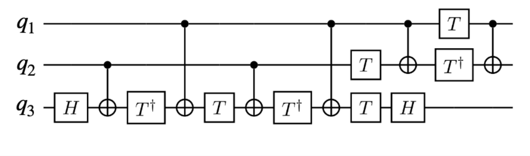



- Rotation shown in the complex plane ($z = x + yi$).
- $r$: Magnitude, $\phi$: Phase.

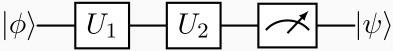
# Quantum Computer Program

- Most commonly shown as a sequence of one and two Qubit gates




- Scores are read from the left to right, But written right to left

$$|\psi\rangle = U_2|U_1\,|\phi\rangle$$




## Common Single Qubit Gates

| Operator | Gate(s) | Matrix |
|---|---|---|
| Pauli-X (X) | X | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| Pauli-Y (Y) | Y | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ |
| Pauli-Z (Z) | Z | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| Hadamard (H) | H | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ |
| Phase (S, P) | S | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ |
| $\pi/8$ (T) | T | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$ |

## Common Two-Qubit Gate

| | | |
|---|---|---|
| Controlled Not (CNOT, CX) | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ |

# The Challenge of Quantum Computing

- What useful computations can you do with unitary transforms on exponentially large matrices?
- Only a few great answers... `So Far!`
  - Simulate Quantum Systems (solve Schrödinger Equation)
  - Factor Integers (Shor's Algorithm)
  - Unstructured Search (Grover's Algorithm)
  - Solving Linear Systems of Equations (HHL)
  - Simulating Large Collections of Oscillating Springs (arXiv:2303.13012)

An Excellent Resource: "`Quantum Algorithm Implementations for Beginners`"
ArXiv:1804.03719

# Questions on Part I?

# Part II: Demystifying Quantum Power Flow