Bayesian Learning Applications in Power System Operations

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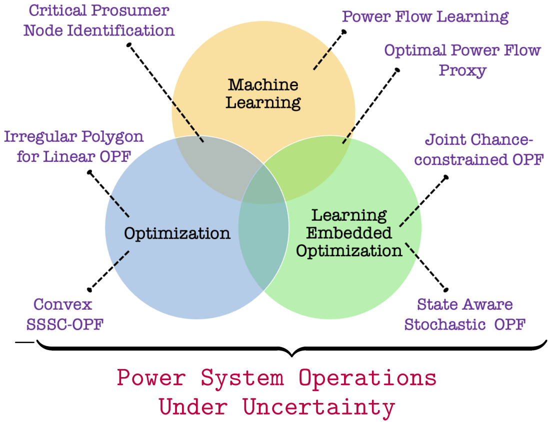
Research Interests: $\mathcal{P}^2$-LAB



- Machine Learning

$$y = \mathcal{M}(\mathbf{x})$$

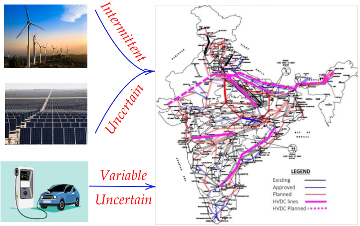
- Optimization

$$\min c(\mathbf{x})$$
$$s.t.\ g(\mathbf{x}, y) \leq 0$$

- Learning-Embedded Optimization

$$\min c(\mathbf{x})$$
$$s.t.\ g(\mathbf{x}, \mathcal{M}(\mathbf{x})) \leq 0$$

Challenges in Power System Operations due to Characteristics of Renewable Sources & Variable Loads



Network Congestion
Voltage Violations
Price Fluctuations
Spatial Variability
Control Issue

Power System Operator

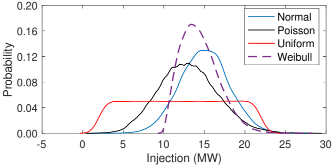
`Technological Advancements`

`Infrastructure Upgrades`

`Optimal Operations`

Happy story on left, becoming a Challenge towards right

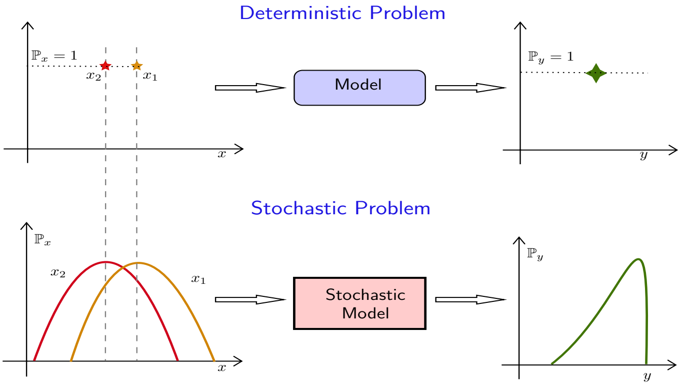
NON-PARAMETRIC UNCERTAINTY: Situation of unknown parameters of probability distributions of random power injections



Distributions in 0-25 MW box

Models & Methods need to follow non-parametric approach for handling uncertainty

Problem and Context



Problem of Risk Estimation & Uncertainty Quantification

- What is the expected value of node voltage violation?
- What are the chances of obtaining node voltage beyond operational limits?

Problem and Context

Net Power Injection

Complex Voltage

Power Flow: $\Longrightarrow$ $$S_i = \sum_{j \in \mathcal{N}} y_{ij}^{\dagger}(v_i v_i^{\dagger} - v_i v_j^{\dagger})$$

Network Parameter

Power flow equation set allows to obtain the complex voltage values at each network node, given the power injection at each node.

Given	Want
Operating Conditions	Expected Violation (Risk)
Load Uncertainty Set	Probability of Violation

- Time & Sample Complexity • Varying Operating Conditions
- Arbitrary Uncertainty Sets – Non-parametric Uncertainties

Why to use ML for power flow learning?

Given		Want
Operating Conditions	Statistical Estimation $\longrightarrow$	Expected Violation (Risk)
Load Uncertainty Set		Probability of Violation

How many Samples ?

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Hoeffding's Inequality

For acceptable error $\varepsilon = 2 \times 10^{-3}$ & 95% confidence, number of samples needed are

$$M \geq 0.5 \log(2/\beta)\varepsilon^{-2} \geq 200,258$$

$$\implies \Big|\text{True Expected Violation } - \text{Estimated Expected Violation}\Big| < 2 \times 10^{-3}$$

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Running Newton-Raphson load flow $200,258$ times is challenging

More so in limited time to quantify 'operational risk'

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Solution $\implies$ A Fast Evaluating ML proxy of Power Flow

How good an ML tool can be for risk estimation?

If NRLF-based statistical estimation have error ε_h, then same number of ML evaluations will achieve $\big|$True Expected Violation $-$ Estimated Expected Violation$\big|$

$$< \underbrace{L}_{\text{Lipschitz Constant}} \overbrace{\varepsilon_m}^{\text{ML Prediction Error}}(1-\overbrace{\delta}^{\text{ML Prediction Confidence}}) + \underbrace{\mathcal{M}}_{\text{Arbitrarily Large Number}}\delta + \overbrace{\varepsilon_h}^{\text{NRLF Estimation Error}}$$

with confidence $1-\beta$, under a condition.

Main Result

Performance using $200,258$ NRLFs $\equiv$ Performance using $801,029$ ML Evaluations
Using Gaussian process (GP) model $L\varepsilon_m(1-\delta) + M\delta + \varepsilon_h \approx 2 \times \varepsilon_h$

Condition: Probability of the absolute difference between true & predicted voltage (by ML) being larger than ε_m is less than or equal to δ.

Proof follows from the fact that with ML performance condition allows to upper bound difference in true and ML based estimation with $L\varepsilon_m(1-\delta) + \mathcal{M}\delta$

Why Gaussian Process (GP) is suitable for Power Flow learning?

GP regression uses **prior knowledge** about a function along with data, to **predict unobserved values** and **quantifying prediction uncertainty**.

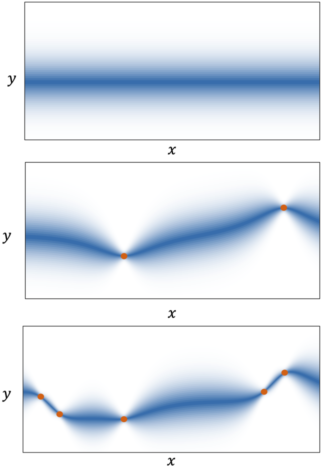
– Provide performance guarantee without requiring ground truth solutions
– Has low training data requirement due to prior knowledge infusion

A Closed-form Power Flow Approximation Framework is

- `Flexible` $\implies$ Non-linear forms with *complexity-accuracy* trade-off
- `Easy to Evaluate` $\implies$ Faster numerical calculations
- `Non-parametric` $\implies$ Works within a power injection range or hypercube
- `Differentiable` $\implies$ Can be fed into optimization problems
- `Interpretable` $\implies$ Should provide insights into physical system

Essentially an **explicit expression** of voltage as a function of power injection which provides a **probabilistic performance guarantee** for risk estimation

GP Working Idea: In Brief



- Prior Knowledge: Smooth Function

$$k(x_i, x_j) = \tau^2 \exp\left\{\frac{-\|x_i - x_j\|_2^2}{2\ell^2}\right\}$$

- Predict unobserved values: Mean

Thickest Shade of Blue

- Prediction uncertainty: Variance

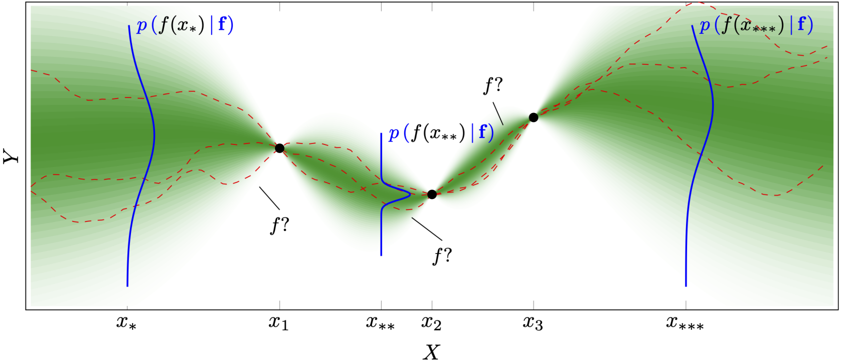
Lighter blue – lower probability

- Performance Guarantee†

$$\mathbb{P}\left\{\left|V(\mathbf{s}) - [\mu(\mathbf{s}) + \gamma\sigma(\mathbf{s})]\right| \geq \varepsilon_m\right\} \leq \delta(\gamma)$$

† Considering the power flow function is a GP.

GP Assumption

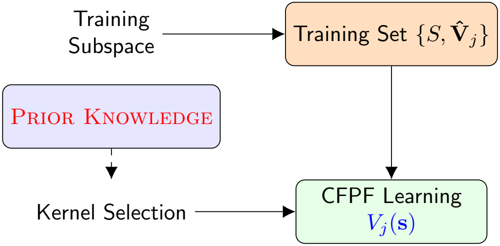


The distribution of $f(x^{\star})$, $f(x^{\star\star})$ and $f(x^{\star\star\star})$ for the three inputs $x^{\star}$, $x^{\star\star}$ and $x^{\star\star\star}$, conditional on the observed values $f(x_1)$, $f(x_2)$ and $f(x_3)$.

Andreas Lindholm *et.al.* "An introduction to Gaussian process regression" Department of Information Technology, Uppsala University.

CFPF Learning: Using Gaussian Process

CFPF Learning Mechanism



Mean : $\mathbb{E}[f(\mathbf{s})] = V_j(\mathbf{s}) = \mathbf{k}^T[K + \sigma_\epsilon^2 I]^{-1}\widehat{\mathbf{V}}_j$

Variance : $\sigma^2[f(\mathbf{s})] = k(\mathbf{s}, \mathbf{s}) - \mathbf{k}^T[K + \sigma_\epsilon^2 I]^{-1}\mathbf{k}$

Training Data– $\{S, \widehat{\mathbf{V}}_j\}$; N Samples

Design Matrix– $S = [\mathbf{s}^1 \ldots \mathbf{s}^i \ldots \mathbf{s}^N]$

– $\mathbf{s}^i$ is i-th power injection vector

Target Vector– $\widehat{\mathbf{V}}_j = [V_j^1 \ldots V_j^N]$

Power Flow as a Function:

$$\widehat{V}_j = f(\mathbf{s}) + \epsilon$$

Gaussian Process (GP) function view†

$$f(\mathbf{s}^i) \sim \mathcal{GP}(\underset{\text{Zero Mean}}{0}, \underset{\text{Kernel Function}}{k}(\mathbf{s}^i, \mathbf{s}^j))$$

Optimize the kernel hyper-parameters using log marginal likelihood

CFPF provides mean prediction of voltage and confidence in that prediction

† C. K. Williams and C. E. Rasmussen, Gaussian processes for machine learning. MIT press Cambridge, MA, 2006, vol. 2, no. 3.

CFPF: Forms

Training Data

$$V_j(\mathbf{s}) = \left[k(\mathbf{s}^1, \mathbf{s}) \quad \dots \quad k(\mathbf{s}^N, \mathbf{s})\right] \boldsymbol{\alpha}_j$$

Variable $\mathbf{s} = [\mathbf{p}; \mathbf{q}]$ Constant

How Do Forms Look Then?

Simple, Standard Forms

– *Linear:* $\mathbf{v} = A\mathbf{s} + b$ – *Quadratic:* $V_j(\mathbf{s}) = \mathbf{s}^T M \mathbf{s} + \mathbf{m}^T \mathbf{s} + r$

More Complex but Accurate Forms

$$V_j(\mathbf{s}) = \sum_{i=1}^{N} \alpha_j(i) \beta^i$$

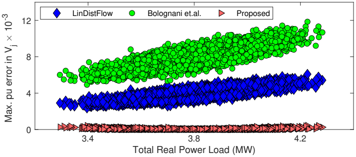
where, $\beta^i = \tau^2 \exp\left(-\|\mathbf{s}^i - \mathbf{s}\|^2 / 2l^2\right)$: Gaussian Kernel

$\mathbf{v}$: Node Voltage Vector; M : Sensitivity Matrix; $\mathbf{m}$: Slope Vector;

$\mathbf{s}$: Node Power Injection Vector

Performance of CFPF)

- Subspace-wise Approximations & Non-Parametric



Error Not a Function of Load

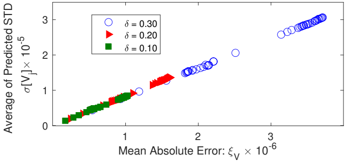
For 69-Bus System

Non-parametric Nature

Distribution	Max. MAE (pu)
Normal	1.86E-05
Beta	2.39E-05
Laplace	2.29E-05
Mixed	1.74E-05

For 33-Bus System

- Inherent Accuracy Indicator



Use of Predictive Variance

For 56-Bus System

- Differentiable Functions
- Faster Numerical Evaluations
- Model Interpretability via *Hyper-parameters*
- Independent from Network Type Assumptions

Graph-Structured Kernel for Low Sample Complexity CFPF Learning

Combining Physics and Intuition into GP

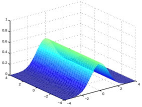
Standard CFPF Learning: Limitations & Solutions

- ✖ **Curse-of-Dimensionality**
  Complexity $\mathcal{O}(N^3)$ with N training samples; Systems $\leq$ 100-Bus
- ✖ **Mesh Network Flows**
  Power injection-voltage relationship is not direct; Higher order nonlinearity
- ✖ **Limited Use**
  Full GP is restrictive to be used within Bayesian Optimization

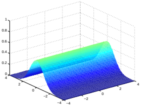
- Learn individual low-dimensional sub-function & combine
  Lower Curse-of-Dimensionality
- Capturing localized voltage-power injection relationship will be easier
  Suitable for Mesh Network Flows
- Low-dimensional GPs
  Useful as surrogate in Bayesian Optimization approach

An Ideal Situation Would be if

A high-dimensional voltage function is breakable into low-dimensional sub-functions

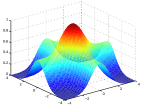


$k_1(x_1, x_1')$
1-D Kernel

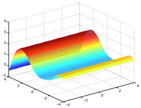


$k_2(x_2, x_2')$
1-D Kernel

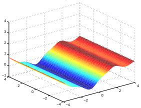
$=$



$k_1(x_1, x_1') + k_2(x_2, x_2')$
Additive Kernel

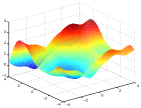


$f_1(x_1)$
1-D GP Prior



$f_2(x_2)$
1-D GP Prior

$=$



$f_1(x_1) + f_2(x_2)$
Additive GP Prior

$$
\begin{aligned}
V_j(\mathbf{s}) &= V_j^1(\mathbf{s}_1) + \cdots + V_j^m(\mathbf{s}_m) \\
&\equiv \mathcal{GP}_1\Big(0, K_1(\mathbf{s}_1, \cdot)\Big) + \cdots + \mathcal{GP}_m\Big(0, K_m(\mathbf{s}_m, \cdot)\Big) \\
&\equiv \texttt{Additive Gaussian Process}
\end{aligned}
$$

Using Network-Structure to Achieve Additive GP Architecture

- Neighboring power injections have highly correlated effect on node voltages
- Effect of far away power injections is approximately equal to sum of individual effects

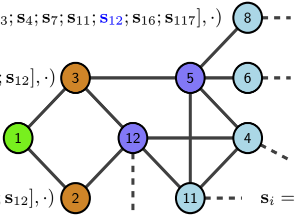
Power Injection Vectors

Number of Nodes

$$k_v(\mathbf{s}^i, \mathbf{s}^j) = \sum_{b=1}^{|\mathcal{B}|} k_b(\mathbf{x}_b^i, \mathbf{x}_b^j)$$

Node Neighborhood Kernel

Neighborhood Aggregated Injection Vectors



Idea of vertex degree kernel construction

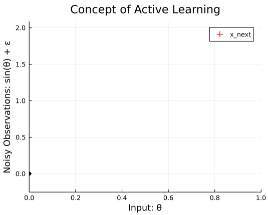
Target Node Voltage

$$V_1(\mathbf{s}) = \underbrace{V_{11}(\mathbf{x}_1) + \cdots + V_{1|\mathcal{B}|}(\mathbf{x}_{|\mathcal{B}|})}_{\text{Latent Node Voltage Functions}}$$

Active Learning: Lower Samples and Stopping Criteria

- ★ What: Learning by successively selecting the next training point 'intelligently'
- ★ Why: To speed-up the learning process using unlabeled data i.e. only input data needed $\implies$ Low Sample Complexity $\implies$ Less Power Flow Samples Needed
- ★ Concept: Next training point is the one which has maximum uncertainty in underlying function

$$\mathbf{s}^{t+1} = \arg\max_{\mathbf{s} \in \mathcal{L}} \sigma_f^t(\mathbf{s}) \rightarrow \texttt{Only Function Evaluation}$$

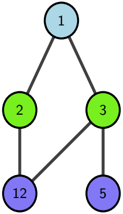


Provides a Proxy of Error ➞ Stopping Criteria

Finding maximum variance point for large-dimensional input space is hard
$\implies$ Used mostly up to 20-dimensions
$\implies$ Power systems have 100s of uncertain power injections

$$\text{Variance:}\quad \sigma^2[f(\mathbf{s})] = \underbrace{\sum_{b=1}^{|\mathcal{B}|} k_b(\mathbf{x_b},\mathbf{x_b})}_{\texttt{Constant}} - \underbrace{\left[\sum_{b=1}^{|\mathcal{B}|} k_b(X_b,\mathbf{x_b})\right]^T}_{\texttt{Variable Vector}} \underbrace{[K_1+\cdots+K_{|\mathcal{B}|}]^{-1}}_{\texttt{Constant Matrix}} \underbrace{\left[\sum_{b=1}^{|\mathcal{B}|} k_b(X_b,\mathbf{x_b})\right]}_{\texttt{Variable Vector}}$$

Neighborhood Aggregated Injection Vectors $\mathbf{x_b}$'s have Overlap



$$\underbrace{[\mathbf{s}_1^{t+1};\mathbf{s}_2^{t+1};\mathbf{s}_3^{t+1}]}_{\widehat{\mathbf{x}}_{\mathcal{D}_1}^{t+1}} = \arg\max_{\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3} \sigma_f^t\left(\widehat{\mathbf{x}}_{\mathcal{D}_1}, \widehat{\mathbf{x}}^t_{\mathcal{D}_{2\ldots d}}\right)$$

$$\underbrace{[\mathbf{s}_5^{t+1};\mathbf{s}_{12}^{t+1}]}_{\widehat{\mathbf{x}}_{\mathcal{D}_2}^{t+1}} = \arg\max_{\mathbf{s}_5,\mathbf{s}_{12}} \sigma_f^t\left(\widehat{\mathbf{x}}^{t+1}_{\mathcal{D}_1}, \widehat{\mathbf{x}}_{\mathcal{D}_2}, \widehat{\mathbf{x}}^t_{\mathcal{D}_{3\ldots d}}\right)$$

. . . .

Idea of network-swipe algorithm for AL.

Algorithm 1 Network-Swipe Algorithm for AL

Require: T, $\mathcal{D}$, $\{\mathbf{s}^1, V^1\}$, $\sigma^2_{threshold}$, T_{max}

Initialize GP model (6) with VDK (9)

while $\sigma_f^2(\mathbf{s}^t) \geq \sigma^2_{threshold}$ **do**

  Solve (11) for $\widehat{\mathbf{x}}^{t+1}_{\mathcal{D}_i}$, sequentially for $i = 1\ldots d$

  Solve ACPF for load $\mathbf{s}^{t+1}$ to get V^{t+1}

  Update GP model with $(\mathbf{s}^{t+1}, V^{t+1})$; $t = t+1$

  If `runtime` $> T_{max}$ **then** `break`

end while

Output: Compute $\mu_f(\mathbf{s}), \sigma_f^2(\mathbf{s})$ for final GP

Benchmarking Performance

Comparison of MAE performance of different methods for 118-Bus System

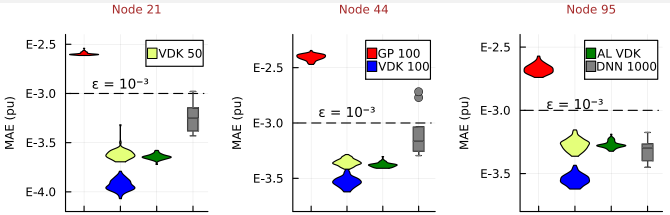


Table: Sample and Time Requirement for Risk Estimation in 500-Bus System

Node	Samples	Time(s)	MAE (pu)$\times 10^{-4}$	ΔVE $\times 10^{-4}$
4	67 - 70	28 - 30	3.11 $\pm$ 1.0	7.8 $\pm$ 0.5
268	102 - 109	53 - 58	4.68 $\pm$ 1.5	7.9 $\pm$ 0.2
320	72 - 76	30 - 33	4.66 $\pm$ 1.8	7.8 $\pm$ 0.4
321	70 - 77	30 - 33	4.96 $\pm$ 2.5	6.8 $\pm$ 0.5
-	Mean evaluation time for 80100 samples is 33.2 sec			

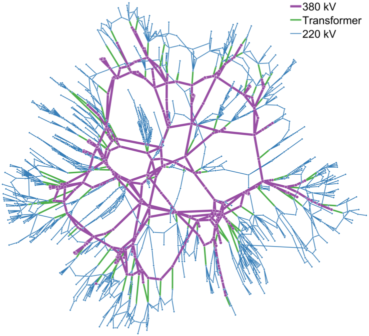
VDK-AL performs the risk estimation for any node in $\approx$ 120 sec

NRLF running time for 20025 samples is $\approx$ 4205 sec

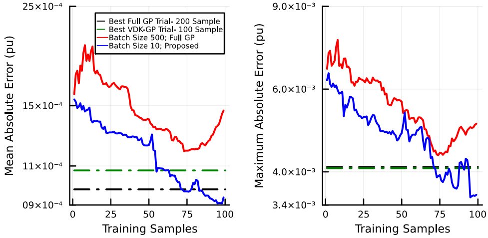
Demonstrating the efficiency and low sample-complexity of AL-VDK & Usefulness for power system operation under uncertainty

Active Learning Performance for 1354-Bus System

- 100 samples are sufficient to learn a node voltage function with 2236 random power injections



1,354-Buses, 260 generators, & 1,991-lines.

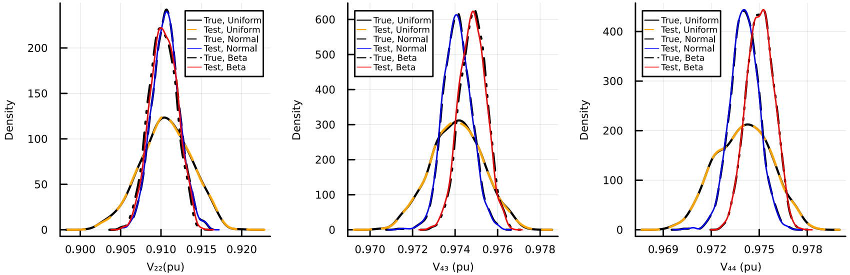


Learning V_5 for 1354-Bus system

	Learning Method (Training Samples)		
	Full GP (200)	VDK-GP (100)	AL (100)
#PF	5000	2500	**100**

A random trial of active learning outperforms large number of passive learning attempts
Reduced time complexity of 25-50 times

Application: Uncertainty Quantification (UQ)



Density functions for UQ in 118-Bus system for node voltages V_{22}, V_{43} & V_{44}

Voltage Violation probability(VP) $\Longrightarrow$ Probability of voltage value being out of limits

Voltage Violation probability (VP) estimation with various input distributions for 33-bus system

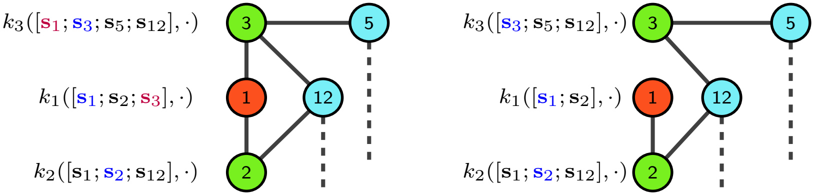
Distribution	Normal	Beta	Laplace	Mixed	Comments
Max. Error in VP	0.0026	0.0028	0.0029	0.0009	$\lvert$Monte-Carlo $-$ Proposed$\rvert$
Time Proposed (s)	$1.04 + 1.66 + 0.70 \times 4 =$ **5.5**				Train Once; Evaluate Multiple
Time MCS (s)	105.2	103.6	100.4	108.3	Each Time 10000 PF Solving

Single model works for different distributions without the need for retraining

Multi-Task Vertex Degree Kernel for CFPF under Network Contingencies

Using ‘Available’ Knowledge to Reduce Training Sample Requirement

Using VDK's Ability to Accommodate Graph Structure & Existing Models to Learn CFPF for 'New' Network Topology



Different sub-kernels of VDK for different topologies of a part of 118-Bus system.

Multi-Task Vertex Degree Kernel (MT-VDK)

Power Injection Vectors

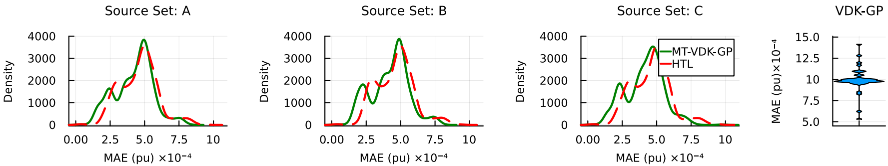
Weight

Number of Source Networks

$$k_{\mathcal{E}_T}(\mathbf{s}^i,\mathbf{s}^j) = \underbrace{\sum_{b=1}^{|\mathcal{B}|} k_b(\mathbf{x}_b^i,\mathbf{x}_b^j)}_{\texttt{VDK on Target } \mathcal{E}_T} + \omega \underbrace{\sum_{m=1}^{M_s} k_{\mathcal{E}_m}(\mathbf{s}^i,\mathbf{s}^j)}_{\texttt{VDKs on Source } \mathcal{E}_m\texttt{'s}}$$

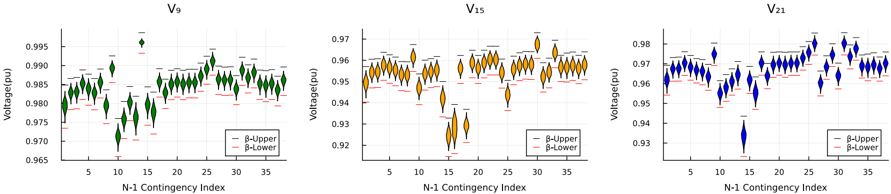
Target Network Graph

More Results



Mean absolute error (MAE) densities for 24 node voltages for 38 different networks (N-1 contingencies).

Average error HALVED using MT-VDK-GP with same number of training sample



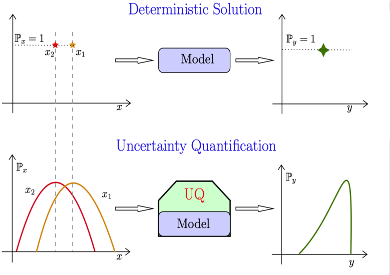
Probabilistic Voltage Envelopes (PVEs) for node voltages & 38 different structures of 30-Bus network

Summary & Applications

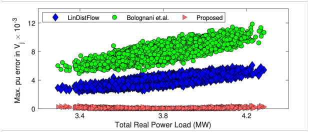
Standard GP → VDK-GP & MT-VDK-GP → Network-Swipe AL

Application Specific Physics-inspired Kernels can help build Interpretable Learning Models in Low Data Regimes.

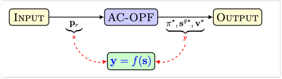
Applications of Physics-inspired Power Flow Learning



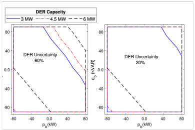
Uncertainty Quantification

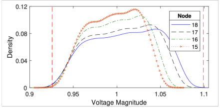


Linear Power Flow under Uncertainty



Closed-form OPF Proxy





Privacy-preserving Feasibility Assessment

All above mentioned works incorporate uncertainties in non-parametric manner

Thank You!

Collaborators

- Dr. Sidhant Misra
- Prof. Lalit Goel
- Dr. L.P.M.I. Sampath
- Dr. Deepjyoti Deka
- Prof. Ashu Verma
- Prof. Hung D. Nguyen
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- Dr. Carlton Coffrin
- Dr. Weng Yu

$\mathcal{P}^2$ LAB

@EE, IIT Roorkee

Our group is currently working on
Machine Learning applications in Power Systems.

We have a few **fully funded PhD positions**
for motivated candidates interested in this area.

In case you know someone who might be a good fit,
please feel free to refer them to us.

More info:*https://psquare-lab.github.io/people/*

ML Performance Bounds

Theorem

Given for an ML model $\mathbb{P}\left\{\left|V(\mathbf{s}^i) - \widehat{V}(\mathbf{s}^i)\right| > \varepsilon_m\right\} \leq \delta$ with $\delta \in (0,1)$ for any $\mathsf{s} \in \mathcal{S}$, and $h(\cdot)$ is a Lipschitz continuous function with Lipschitz constant L, then the error in expected value estimation using the ML model, for any $\mathsf{s} \in \mathcal{S}$, is bounded with probability greater than $1-\beta$ as

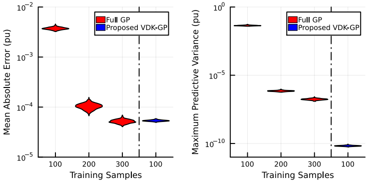
$$\left|\mathbb{E}[h_p(\mathbf{s})] - \widehat{\mathbb{E}}[h_m(\mathbf{s})]\right| \leq L\varepsilon_m(1-\delta) + M\delta + \varepsilon_h$$

where, $\beta \in (0,1)$, $\varepsilon_h = \sqrt{\frac{\log(2/\beta)}{2N}}$, $M > |h_p(\mathbf{s}) - h_m(\mathbf{s})|$, and N number of samples.

$h(\cdot)$ violation function measuring distance from voltage limits

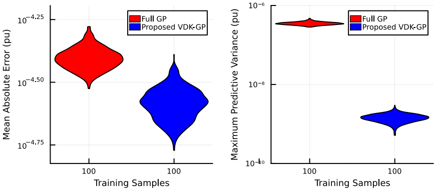
Benchmarking

500 independent trials with 100 training samples; Testing: 1000 unique samples



V_1 within $\pm 10\%$ hypercube for 500-Bus system.

Three Times Lower Sample Complexity



V_1 within $\pm 10\%$ hypercube for 118-Bus system.

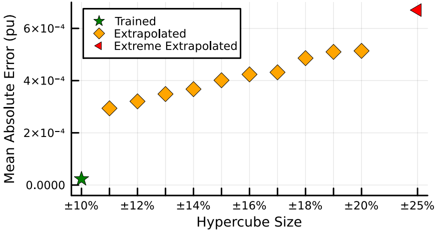
50% Lower Error & 100 Times Confident Model

Proposed VDK-GP *outperforms* a *3-layer, 1000-neuron Deep Neural Network* for using 100 training samples in 118-Bus system

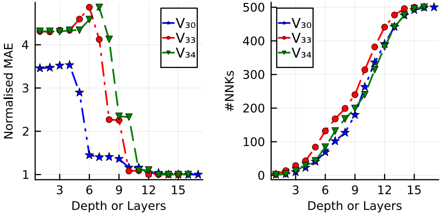
Node	*MAE (pu)*	
	Proposed	*DNN*
1	$\mathbf{5.22 \times 10^{-5}}$	1.89×10^{-4}
43	$\mathbf{8.70 \times 10^{-5}}$	9.77×10^{-4}
117	$\mathbf{2.26 \times 10^{-5}}$	9.05×10^{-4}

Accurate, loading independent power flow model with extremely low sample complexity
Useful for power system operation under uncertainty

More Insights: Extrapolation & Depth Effect

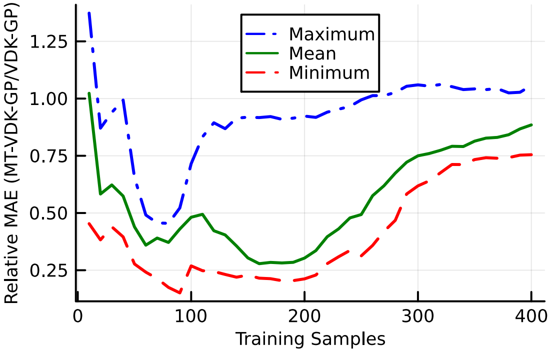


Extrapolation of VDK-GP model trained within $\pm 10\%$ hypercube for 118-Bus system.



Effect of depth on learning quality of three different voltage function in 500-Bus system.

VDK-GP V/S MT-VDK-GP



Relative MAE variation with respect to training samples– defined as the ratio of MAE obtained using MT-VDK-GP to VDK-GP.

Valleys in the relative error curves reflect the superior performance of the proposed MT-VDK-GP at low data regimes.