Severity-Aware N-1 Secure Chance-Constrained AC OPF

Model scope and event timeline. Let $\mathcal{K} = \{0\} \cup \mathcal{K}^{N-1}$ contain the base state and all non-islanding single generator and line outages. The decision variables are the preventive base-state dispatch

$$x^0 = (\theta^0, v^0, p_G^0, q_G^0),$$

upward/downward corrective reserves $r_i^+, r_i^- \geq 0$, and, for each active contingency $k \in \mathcal{A} \subseteq \mathcal{K}^{N-1}$, the post-contingency steady state

$$x^k = (\theta^k, v^k, p_G^k, q_G^k).$$

The chronology is fixed as follows: (i) choose x^0 and reserve procurement (r^+, r^-); (ii) if contingency k occurs, deterministic corrective redispatch moves the system from x^0 to x^k within the procured reserves; (iii) residual net-injection forecast error $\omega \sim \mathcal{N}(0, \Sigma)$ acts around x^k and is balanced by fixed contingency-dependent response parameters (α^k, γ^k) and local AVR. Thus x^k is a post-contingency, post-corrective, pre-uncertainty operating point. The balancing parameters satisfy $\alpha_i^k = 0$ for outaged generators and $\sum_{i \in \mathcal{G} \setminus \mathcal{G}_k^{\text{out}}} \alpha_i^k = 1$; if the nominal slack is outaged, a distributed-slack implementation based on α^k is used.

Deterministic AC final problem. For each k, let $f^k(x^k) = 0$ denote the full AC power flow equations with the topology and generator availability of contingency k; for line outages, the outaged line is removed from the admittance matrix, and for generator outages, $p_{G,i}^k = q_{G,i}^k = 0$ for the outaged unit. The deterministic SCOPF is

$$\begin{aligned}
\min_{x^0, \{x^k\}_{k \in \mathcal{A}}, r^+, r^-} \quad & C(p_G^0) + \sum_{i \in \mathcal{G}} (c_i^+ r_i^+ + c_i^- r_i^-) && && (1)\\
\text{s.t.} \quad & f^0(x^0) = 0, && && (2)\\
& f^k(x^k) = 0, && k \in \mathcal{A}, && (3)\\
& -r_i^- \leq p_{G,i}^k - p_{G,i}^0 \leq r_i^+, && i \in \mathcal{G} \setminus \mathcal{G}_k^{\text{out}},\ k \in \mathcal{A}, && (4)\\
& 0 \leq r_i^+ \leq \bar{r}_i^+, \qquad 0 \leq r_i^- \leq \bar{r}_i^-, && i \in \mathcal{G}. && (5)
\end{aligned}$$

Equation (4) makes post-contingency corrective redispatch explicit; without it, the contingency states are not operationally meaningful.

State-specific stochastic linearization. Each scalar limit is written as an upper-bound inequality. For every scalar constraint c in state k, we define the monitored quantity $z_{c,k}(x^k)$ and its local stochastic approximation

$$\tilde{z}_{c,k}(\omega) = z_{c,k}(x^k) + b_{c,k}(x^k)^\top \omega, \qquad \sigma_{c,k} = \|\Sigma^{1/2} b_{c,k}(x^k)\|_2.$$

The sensitivity vector $b_{c,k}$ is recomputed from the contingency-specific AC Jacobian and the fixed policy (α^k, γ^k) at every active state x^k. Weighted voltage constraints are imposed only at buses modeled as PQ in state k.

Hard constraints. Let $\mathcal{C}_H$ denote generator active- and reactive-power capability constraints. These are kept as ordinary chance constraints,

$$\mathbb{P}(\tilde{z}_{c,k}(\omega) \leq \bar{z}_{c,k}) \geq 1 - \varepsilon_{c,k}, \qquad c \in \mathcal{C}_H,$$

with deterministic equivalent

$$z_{c,k}(x^k) \leq \bar{z}_{c,k} - \lambda_{c,k}^H, \qquad \lambda_{c,k}^H = \Phi^{-1}(1 - \varepsilon_{c,k})\, \sigma_{c,k}.$$

Soft constraints. Let $\mathcal{C}_S$ denote line-current upper limits and PQ-bus voltage upper/lower limits. For each $c \in \mathcal{C}_S$, we specify a normal limit $\bar{z}_{c,k}^N$, an emergency limit $\bar{z}_{c,k}^E > \bar{z}_{c,k}^N$, a normalization scale $\Delta_c > 0$, a weighted-risk budget τ_c, and an emergency violation probability $\bar{\varepsilon}_c$. We define the signed excess above the normal limit as

$$Y_{c,k}(\omega) = \tilde{z}_{c,k}(\omega) - \bar{z}_{c,k}^N \sim \mathcal{N}(\mu_{c,k}, \sigma_{c,k}^2), \qquad \mu_{c,k} = z_{c,k}(x^k) - \bar{z}_{c,k}^N.$$

The soft constraint is enforced by the hybrid pair

$$\mathbb{E}\left[\psi_c\left(\frac{[Y_{c,k}(\omega)]_+}{\Delta_c}\right)\right] \leq \tau_c, \tag{6}$$

$$\mathbb{P}\Big(\tilde{z}_{c,k}(\omega) \leq \bar{z}_{c,k}^E\Big) \geq 1 - \bar{\varepsilon}_c. \tag{7}$$

We use the convex piecewise-linear severity model

$$\psi_c(t) = \sum_{m=0}^{M_c} \eta_{c,m}[t - h_{c,m}]_+, \qquad h_{c,0} = 0, \quad 0 = h_{c,0} < h_{c,1} < \cdots < h_{c,M_c}, \quad \eta_{c,m} \geq 0.$$

For $Y \sim \mathcal{N}(\mu, \sigma^2)$, we define

$$r_a(\mu, \sigma) := \mathbb{E}[(Y - a)_+] = \sigma\varphi(\beta_a) + (\mu - a)\Phi(\beta_a), \qquad \beta_a = \frac{\mu - a}{\sigma}.$$

Then the weighted risk in (6) is available in closed form:

$$\mathcal{R}_c(\mu, \sigma) := \mathbb{E}\left[\psi_c\left(\frac{[Y]_+}{\Delta_c}\right)\right] = \frac{1}{\Delta_c}\sum_{m=0}^{M_c} \eta_{c,m}\, r_{\Delta_c h_{c,m}}(\mu, \sigma).$$

Since $\partial r_a / \partial \mu = \Phi((\mu - a)/\sigma) \in (0, 1)$, $\mathcal{R}_c(\mu, \sigma)$ is continuous and strictly increasing in μ. Hence, for every $\sigma \geq 0$, the weighted margin

$$\lambda^W_{c,k}(\sigma_{c,k}) := \inf\left\{\lambda \geq 0 : \mathcal{R}_c(-\lambda, \sigma_{c,k}) \leq \tau_c\right\}$$

is well-defined and unique. Therefore (6)–(7) are equivalent to the deterministic tightenings

$$z_{c,k}(x^k) \leq \bar{z}^N_{c,k} - \lambda^W_{c,k}(\sigma_{c,k}), \qquad z_{c,k}(x^k) \leq \bar{z}^E_{c,k} - \Phi^{-1}(1 - \bar{\varepsilon}_c)\sigma_{c,k}.$$

Complete upgrade problem. The final optimization is (1)–(5) plus, for every active contingency $k \in \mathcal{A}$, all deterministic equivalents of the hard and soft constraints above. The scalable solution method is a two-level scheme: for fixed active set $\mathcal{A}$, solve the deterministic AC SCOPF with current margins, recompute $b_{c,k}$ and all margins, and iterate to a fixed point. Then update the active set by

$$\mathcal{A}^{(t+1)} = \mathcal{A}^{(t)} \cup \Big\{k \in \mathcal{K}^{N-1} \setminus \mathcal{A}^{(t)} : \exists c \text{ such that a hard, weighted-normal, or emergency deterministic equivalent is violated in state } k\Big\}.$$

Our model does not claim global optimality or convergence of the outer-inner algorithm; it only claims a precise deterministic equivalent of the chosen local stochastic approximation and an exact AC validation target for every active contingency state.