Chance-Constrained OPF in Microgrids: A Data-Driven Distributionally Robust Approach with Droop Control and Power Flow Routers

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I. System Model

Consider an islanded microgrid with n buses, line set $\mathcal{E}$, dispatchable distributional generator (DG) set $\mathcal{N}_G$, and renewable DG set $\mathcal{N}_W$. Dispatchable DGs employ P-ω / Q-V droop control:

$$\omega(\boldsymbol{\xi}) = \omega^* - K_{p_i}\left(P_{G_i}(\boldsymbol{\xi}) - P^*_{G_i}\right), \quad \forall i \in \mathcal{N}_G, \tag{1}$$

$$V_i(\boldsymbol{\xi}) = V_i^* - K_{q_i}\left(Q_{G_i}(\boldsymbol{\xi}) - Q^*_{G_i}\right), \quad \forall i \in \mathcal{N}_G. \tag{2}$$

Renewable power: $P_{W_i}(\boldsymbol{\xi}) = P^f_{W_i} + \xi_i,\ Q_{W_i} = \lambda_i P_{W_i}$. For lines with power flow routers (PFRs) (tap T^*_{ij}, phase β^*_{ij}):

$$P^{\mathrm{PFR}}_{ij} = g_{ij}\left(T^{*2}_{ij}V_i^2 - T^*_{ij}T^*_{ji}V_iV_j\cos(\theta_{ij}+\beta^*_{ij})\right) - b_{ij}T^*_{ij}T^*_{ji}V_iV_j\sin(\theta_{ij}+\beta^*_{ij}), \tag{3}$$

$$Q^{\mathrm{PFR}}_{ij} = -b_{ij}\left(T^{*2}_{ij}V_i^2 - T^*_{ij}T^*_{ji}V_iV_j\cos(\theta_{ij}+\beta^*_{ij})\right) - g_{ij}T^*_{ij}T^*_{ji}V_iV_j\sin(\theta_{ij}+\beta^*_{ij}). \tag{4}$$

II. DR-CC-OPF-PFR Formulation

Rather than assuming $\boldsymbol{\xi} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$, we construct a Wasserstein ambiguity set from N historical samples $\{\boldsymbol{\xi}^{(s)}\}_{s=1}^N$:

$$\mathcal{P}_W = \left\{P : \mathcal{W}_1(P, \hat{P}_N) \le r\right\}, \quad r = C \cdot N^{-\frac{1}{2}}\sqrt{\log\frac{1}{\alpha}}, \tag{5}$$

where $\hat{P}_N = \frac{1}{N}\sum_s \delta_{\boldsymbol{\xi}^{(s)}}$ is the empirical distribution and r is chosen so that the true distribution lies in $\mathcal{P}_W$ with probability $1-\alpha$. The proposed DR-CC-OPF-PFR problem is:

$$\min \quad \mathbb{E}\left[\sum_{i\in\mathcal{N}_G} c_{2i}P_{G_i}(\boldsymbol{\xi})^2 + c_{1i}P_{G_i}(\boldsymbol{\xi}) + c_{0i}\right] \tag{6a}$$

(6b)

$$\text{s.t.} \quad P_{G_i}(\boldsymbol{\xi}) + P_{W_i}(\boldsymbol{\xi}) - P_{L_i} = \sum_{(i,j)\in\mathcal{E}} P^{\mathrm{PFR}}_{ij}(\boldsymbol{V}(\boldsymbol{\xi}), \boldsymbol{\theta}(\boldsymbol{\xi}), \boldsymbol{T}^*, \boldsymbol{\beta}^*),\ \forall i \in \mathcal{N} \tag{6c}$$

$$Q_{G_i}(\boldsymbol{\xi}) + Q_{W_i}(\boldsymbol{\xi}) - Q_{L_i} = \sum_{(i,j)\in\mathcal{E}} Q^{\mathrm{PFR}}_{ij}(\boldsymbol{V}(\boldsymbol{\xi}), \boldsymbol{\theta}(\boldsymbol{\xi}), \boldsymbol{T}^*, \boldsymbol{\beta}^*),\ \forall i \in \mathcal{N} \tag{6d}$$

$$\omega(\boldsymbol{\xi}) = \omega^* - K_{p_i}\left(P_{G_i}(\boldsymbol{\xi}) - P^*_{G_i}\right),\ \forall i \in \mathcal{N}_G \tag{6e}$$

$$V_i(\boldsymbol{\xi}) = V_i^* - K_{q_i}\left(Q_{G_i}(\boldsymbol{\xi}) - Q^*_{G_i}\right),\ \forall i \in \mathcal{N}_G \tag{6f}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(P_{G_i}(\boldsymbol{\xi}) \le P^{\max}_{G_i}\right) \ge 1-\varepsilon_P,\ \forall i \in \mathcal{N}_G \tag{6g}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(P_{G_i}(\boldsymbol{\xi}) \ge P^{\min}_{G_i}\right) \ge 1-\varepsilon_P,\ \forall i \in \mathcal{N}_G \tag{6h}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(Q_{G_i}(\boldsymbol{\xi}) \le Q^{\max}_{G_i}\right) \ge 1-\varepsilon_Q,\ \forall i \in \mathcal{N}_G \tag{6i}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(Q_{G_i}(\boldsymbol{\xi}) \ge Q^{\min}_{G_i}\right) \ge 1-\varepsilon_Q,\ \forall i \in \mathcal{N}_G \tag{6j}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(V_i(\boldsymbol{\xi}) \le V_i^{\max}\right) \ge 1-\varepsilon_V,\ \forall i \in \mathcal{N} \tag{6k}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(V_i(\boldsymbol{\xi}) \ge V_i^{\min}\right) \ge 1-\varepsilon_V,\ \forall i \in \mathcal{N} \tag{6l}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(\omega(\boldsymbol{\xi}) \le \omega^{\max}\right) \ge 1-\varepsilon_\omega \tag{6m}$$

$$\inf_{P\in\mathcal{P}_W} \mathbb{P}\left(\omega(\boldsymbol{\xi}) \ge \omega^{\min}\right) \ge 1-\varepsilon_\omega \tag{6n}$$

$$P^{\min}_{G_i} \le P^*_{G_i} \le P^{\max}_{G_i},\ Q^{\min}_{G_i} \le Q^*_{G_i} \le Q^{\max}_{G_i},\ \forall i \in \mathcal{N}_G \tag{6o}$$

$$V_i^{\min} \le V_i^* \le V_i^{\max},\ \forall i \in \mathcal{N}; \quad \omega^{\min} \le \omega^* \le \omega^{\max} \tag{6p}$$

$$\gamma^{\min}_{ij} \le T^*_{ij} \le \gamma^{\max}_{ij},\ \beta^{\min}_{ij} \le \beta^*_{ij} \le \beta^{\max}_{ij},\ \forall (i,j) \in \mathcal{E} \tag{6q}$$

$$\theta_1 = 0. \tag{6r}$$

TABLE I
Difference between Gaussian CC-OPF and DR-CC-OPF-PFR

	Gaussian CC-OPF	**DR-CC-OPF-PFR**
Uncertainty	$\boldsymbol{\xi} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$	$P \in \mathcal{P}_W$, data-driven
Distribution-free	No	Yes
Conservatism	Fixed	$\downarrow$ as $N \to \infty$

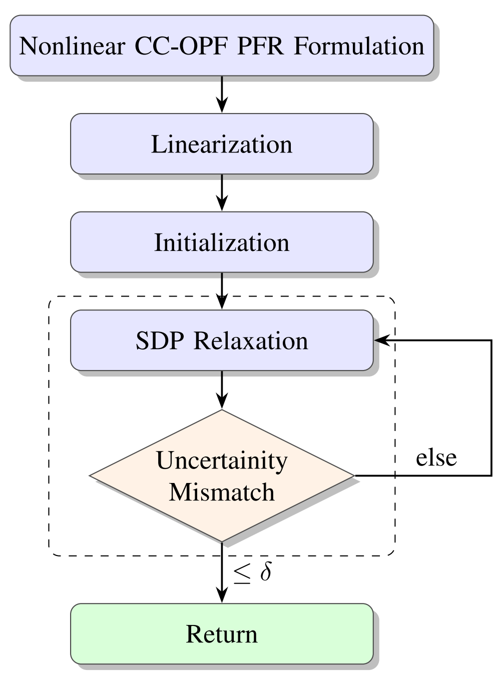

Fig. 1. Iterative SDP-based solution methodology.