Mathematical Formulation: Stability Constrained AC-OPF Using DynOPF-Net

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I. Notation

- N: Buses, $\mathcal{L}$: Lines, $\mathcal{G}$: Generators
- V_i, θ_i: Voltage Magnitude, Phase
- S_{ri}, S_{di}, S_{ij}: Generated, Demand, Line power
- Y_{ij}: Line Admittance
- δ_g, ω_g: Rotor Angle, Speed
- ω_s: Synchronous Frequency
- m_g, d_g: Inertia, Damping constant
- x'_{gd}, e'_{gq}: Machine Parameters
- δ_{max}: Stability Limit
- ψ, ϕ: DynOPF-Net parameters

II. AC Optimal Power Flow

Power definitions:

$$S_{ri} = p_{ri} + jq_{ri}, \quad S_{di} = p_{di} + jq_{di} \tag{1}$$
$$S_{ij} = V_i I_{ij}^* \tag{2}$$

Objective:

$$\min \sum_{i \in \mathcal{G}} c_{2i}(Re(S_{ri}))^2 + c_{1i}Re(S_{ri}) + c_{0i} \tag{3}$$

Operational constraints:

$$v_i^l \leq |V_i| \leq v_i^u \tag{4}$$
$$-\theta_{ij}^{\Delta} \leq \angle(V_i V_j^*) \leq \theta_{ij}^{\Delta} \tag{5}$$
$$S_{ri}^l \leq S_{ri} \leq S_{ri}^u \tag{6}$$
$$|S_{ij}| \leq s_{ij}^u \tag{7}$$
$$S_{ri} - S_{di} = \sum S_{ij} \tag{8}$$
$$S_{ij} = Y_{ij}^* |V_i|^2 - Y_{ij}^* V_i V_j^* \tag{9}$$
$$\theta_{ref} = 0 \tag{10}$$

III. Generator Dynamics

$$\frac{d}{dt}\begin{bmatrix}\delta_g \\ \omega_g\end{bmatrix} = \begin{bmatrix}\omega_s(\omega_g - \omega_s) \\ \frac{1}{m_g}\left(p_g^m - d_g(\omega_g - \omega_s) - \frac{e'_{gq}|V_g|}{x'_{gd}}\sin(\delta_g - \theta_g)\right)\end{bmatrix} \tag{11}$$

Initial conditions:

$$\frac{e'_{gq}|V_g|\sin(\delta_g - \theta_g)}{x'_{gd}} - p_{rg} = 0 \tag{12}$$
$$\frac{e'_{gq}|V_g|\cos(\delta_g - \theta_g) - |V_g|^2}{x'_{gd}} - q_{rg} = 0 \tag{13}$$
$$\omega_g(0) = \omega_s \tag{14}$$

Stability constraint:

$$\delta_g(t) \leq \delta_{max} \tag{15}$$

IV. Neural ODE Learning

$$\min_{\phi} \mathbb{E}_{(x_0, x(t))} \|\hat{x}(t) - x(t)\|^2 \tag{16}$$
$$\hat{x}(t) = ODEsolver(\mathcal{N}_\phi, x_0, \Delta t) \tag{17}$$
$$x(t) = \text{ODEsolver}(p, x_0, \Delta t) \tag{18}$$

V. DynOPF-Net Loss

$$\mathcal{L}_{DynOPF} = \mathcal{L}_p + \mathcal{L}_c \tag{19}$$

Prediction loss:

$$\mathcal{L}_p = \|\hat{y} - y^\star\|^2 \tag{20}$$

Constraint penalty:

$$\mathcal{L}_c = \sum \lambda_{h_j} \nu(h_j) + \sum \lambda_{u_l} \nu(u_l) \tag{21}$$

VI. Parameter Updates

$$\psi^{k+1} = \psi^k - \eta \nabla_\psi \mathcal{L}_{DynOPF} \tag{22}$$
$$\phi^{k+1} = \phi^k - \eta \nabla_\phi \mathcal{L}_{DynOPF} \tag{23}$$

Multiplier updates:

$$\lambda_h^{k+1} = \lambda_h^k + \rho \nu(h) \tag{24}$$
$$\lambda_u^{k+1} = \lambda_u^k + \rho \nu(u) \tag{25}$$