Quiz 2: Load Dispatch

Course: EET 110 | **Instructor:** Parikshit Pareek

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The Engineering Scenario

You are the Operations Engineer at the **IIT Roorkee Smart Micro-Grid Controller**. Your task is to supply a critical campus load of exactly **120 MW** using two available sources: Local Distributed Generation (x_1) and Grid Import (x_2). Due to physical infrastructure limits, the local generation cannot exceed 100 MW, and the transmission lines for grid import are rated for a maximum of 80 MW. Both sources must provide non-negative power.

The operational economics of this dispatch are highly uncertain due to fluctuating weather and market conditions. Your technical team has identified three probable scenarios for the upcoming cycle. Under the *Optimal Weather Scenario*, the cost is determined by a baseline of 15 units per MW of local generation and 25 units per MW of grid import, but the system receives a flat environmental rebate of 400 units. In the *Market Volatility Scenario*, high external demand spikes the grid import price to 50 units per MW, while local generation remains relatively efficient at 10 units per MW, though it incurs a fixed operational overhead of 900 units. Finally, under the *Emergency Maintenance Scenario*, local units require specialized cooling, driving local costs up to 70 units per MW, whereas the grid provides subsidized relief at only 5 units per MW, despite a fixed stability tax of 1,200 units imposed on the connection.

As a risk-averse decision-maker, your goal is to determine the power allocation (x_1, x_2) that **minimizes the maximum possible cost** among these three scenarios. This ensures that regardless of which scenario manifests, the fiscal impact is kept to the absolute minimum possible.

1. Task

1. Identify the decision variables and the specific linear cost functions f_1, f_2, and f_3 described in the text.
2. Express the minimax objective function.
3. Transform the problem into a standard Linear Programming (LP) format using an auxiliary variable Z to represent the "worst-case" cost.
4. List all constraints in the form $Ax \leq b$ or $Ax = b$.

Solution and Formulation

1. Cost Function Extraction

Let x_1 be the Local Generation (MW) and x_2 be the Grid Import (MW). Based on the verbal description:

- **Scenario 1 (Optimal):** $f_1(x_1, x_2) = 15x_1 + 25x_2 - 400$
- **Scenario 2 (Market):** $f_2(x_1, x_2) = 10x_1 + 50x_2 + 900$
- **Scenario 3 (Emergency):** $f_3(x_1, x_2) = 70x_1 + 5x_2 + 1200$

2. Minimax Objective

The objective is to minimize the upper envelope of these three functions:

$$\text{Minimize } [\max\{f_1(x_1, x_2), f_2(x_1, x_2), f_3(x_1, x_2)\}]$$

3. Standard LP Transformation

We introduce an auxiliary variable $Z \in \mathbb{R}$ such that Z is greater than or equal to the cost in every scenario.

- **Objective:**
  Minimize Z
- **Subject to (Scenario Constraints):**

$$\begin{aligned} 15x_1 + 25x_2 - Z &\leq 400 \\ 10x_1 + 50x_2 - Z &\leq -900 \\ 70x_1 + 5x_2 - Z &\leq -1200 \end{aligned}$$

- **Subject to (Physical Constraints):**

$$\begin{aligned} x_1 + x_2 &= 120 && \text{(Demand Balance)} \\ x_1 &\leq 100 && \text{(Local Capacity)} \\ x_2 &\leq 80 && \text{(Grid Capacity)} \\ x_1, x_2, Z &\geq 0 && \text{(Non-negativity)} \end{aligned}$$