Labwork 8: Robust LP

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General Instructions:

- You may use standard optimization modeling languages such as `CVXPY` (in Python) or `JuMP` (in Julia) along with a solver like `Gurobi`. `Gurobi` has free academic license.
- For every plot, ensure axes are labeled properly and include legends where multiple curves are shown. I know we do not submit Lab works, but it is important to learn good practices.
- Task marked with **[Handwritten]** must be done using handwritten derivations. It is important to learn.
- I used Claude to make this, of course with a lot of brainstorming. You can use AI, for brainstorming not for zero-shot assignment solving.
- I would follow this pattern: read question → read relevant class notes/slides *to* clear doubts before starting to code via AI or Piazza Peers → start coding with agents → 100% clear in math and whatever you coded → Think what should be the output of your code and why → Get/or Not get that output you wished → Analysis
- You must ensure that you know what you are doing. Try it, you will feel good.

Part 2: Robust Linear Programming Under Uncertainty (Shield Your LP)

Objective: Reformulate a linear program with uncertain constraint coefficients into a tractable robust optimization problem, comparing box and ball uncertainty sets.

Task 2.1: Nominal LP Setup: Consider the following production planning LP with $n = 4$ products:

$$\max_{x\in\mathbb{R}^4} c^T x \quad \text{s.t.} \quad \bar{a}_i^T x \le b_i, \;\; i = 1, \dots, 6, \quad x \ge 0.$$

Use the following data (provided as matrices):

$$\bar{A} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 3 & 2 & 1 \\ 0 & 2 & 4 & 1 \\ 3 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 20 \\ 30 \\ 25 \\ 18 \\ 12 \\ 15 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 7 \end{bmatrix}.$$

Solve the nominal LP and report the optimal x^* and $c^T x^*$.

Task 2.2: Uncertainty Model: Now suppose each constraint coefficient is uncertain: $a_i = \bar{a}_i + P_i u_i$, where $P_i \in \mathbb{R}^{n\times n}$ is a perturbation matrix and u_i is an unknown perturbation vector. Use $P_i = 0.15 \cdot \text{diag}(\bar{a}_i)$ for all i (i.e., each coefficient can vary by ±15% of its nominal value, and the perturbation is axis-aligned).

The robust constraint requires: $a_i^T x \le b_i$ must hold for **all** admissible u_i.

Task 2.3: Comparison and Visualization:

- Create a bar chart comparing $c^T x^*$ for the three cases (nominal, box-robust, ball-robust).
- The ball uncertainty set $\|u\|_2 \le 1$ is contained inside the box $\|u\|_\infty \le 1$ in $\mathbb{R}^n$. Explain why this means the ball-robust solution is *less conservative* (higher objective) than box-robust.
- Pick any two constraints (say $i = 1, 3$). For the nominal, box-robust, and ball-robust optimal solutions, compute the *slack* $s_i = b_i - \bar{a}_i^T x^*$ and plot a grouped bar chart. Interpret the pattern you observe.