Algorithms for Power Grid

Spring 2025-26 — Power System Optimization

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Lecture 1: Introduction to Optimization

What is Optimization?

Optimization is the formal process of making the **best** choice among a set of available alternatives.

Components of Optimization Problem

$$\begin{aligned} \min \quad & c(\mathbf{x}) \\ \text{s.t.} \quad & f(\mathbf{x}) \leq 0 \end{aligned}$$

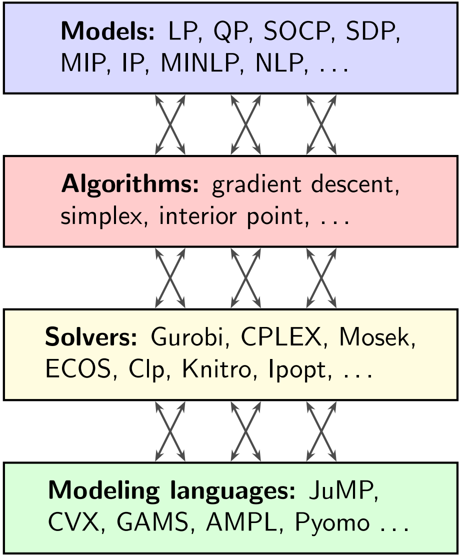
- **Decision Variables:** What can we control? – $\mathbf{x}$
- **Constraints:** What are our limits? (Physics, Budget, Logic)
- **Objective Function:** What are we trying to maximize/minimize? – $c(\mathbf{x})$

Grid Context: Dispatching generators to meet load at minimum cost while respecting line limits.

Real-World Examples

- **Processor Scheduling:** Adjusting speed s_t over time to minimize energy while meeting job deadlines.
- **Environmental Management:** Dam gate scheduling to maximize irrigation water supply while meeting power generation needs.
- **Compressive Sensing:** Recovering signals from under-determined systems (sparse solutions).
- **Power Networks:** Unit commitment and Economic Dispatch.

The Optimization Hierarchy



- **Models:** The mathematical structure (DNA) of the problem.
- **Algorithms:** The iterative logic/recipe to find the optimum.
- **Solvers:** Specialized software implementing the algorithms.
- **Languages:** The interface (e.g., Julia/JuMP) to communicate with solvers.

Taken from Prof. Laurent Lessard's University of Wisconsin–Madison Course.

Speed vs. Generality

Model Type	Generality	Typical Speed
Linear (LP)	Specific	Extremely Fast
Conic (SOCP)	Medium	Moderate
Nonlinear (NLP)	High	Slow / Local Optima
Mixed-Integer (MIP)	Very High	Potentially Exponential

Pro-tip: Choose the least general model that fits your physics. Don't use an NLP solver for an LP problem!

The Jalandhar Sports Gear (JSG) Case Study

Problem Statement: Jalandhar Sports Gear (JSG) is a premier equipment manufacturer in Punjab currently planning its seasonal production for high-grade Cricket Bats and Professional Hockey Sticks. Based on current market data, each Cricket Bat sold yields a profit of ₹1,200, while each Hockey Stick generates a profit of ₹900. The production process is limited by several critical resource constraints: each Cricket Bat requires 4 units of high-grade willow wood and 1 hour of machine polishing, whereas each Hockey Stick requires 2 units of willow wood and 1 hour of machine time. For the upcoming production cycle, the factory has access to a total of 4,800 units of willow wood and a maximum machine capacity of 1,750 hours. Furthermore, due to current supply chain bottlenecks, the warehouse only contains 1,000 rubber grips for bats and 1,500 grips for hockey sticks. Assuming that the company can sell every unit it produces, management must determine the exact optimal quantity of bats and sticks to manufacture this season in order to maximize the total profit.

Case Study: Jalandhar Sports Gear (JSG)

Jalandhar is a global hub for sports manufacturing. JSG produces high-quality wooden gear.

- **Cricket Bats (x_1):**
  Profit: **₹1,200** per unit.
  Requires: 4 units of Willow wood, 1 machine hour.
- **Hockey Sticks (x_2):**
  Profit: **₹900** per unit.
  Requires: 2 units of Willow wood, 1 machine hour.

Current Availability (Constraints):

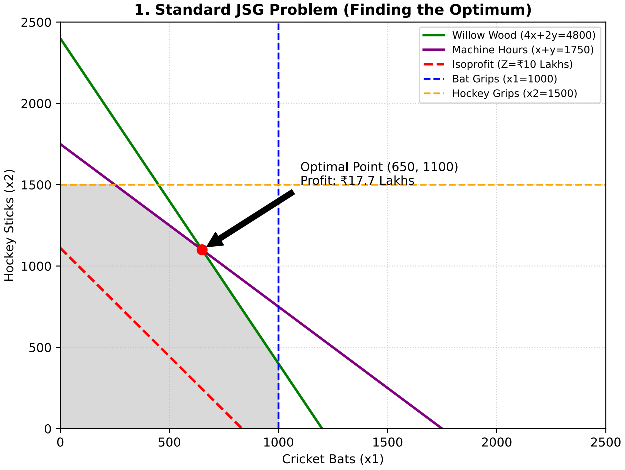
- Total Willow Wood: **4,800 units**.
- Total Machine Capacity: **1,750 hours**.
- Specific Bat Grips in stock: **1,000**.
- Specific Hockey Grips in stock: **1,500**.

Mathematical Formulation

Decision Variables
$x_1 =$ No. of Cricket Bats, $x_2 =$ No. of Hockey Sticks

$$
\begin{aligned}
\max \quad & 1200x_1 + 900x_2 && \text{(Total Profit)} \\
\text{s.t.} \quad & 4x_1 + 2x_2 \le 4800 && \text{(Willow Wood Limit)} \\
& x_1 + x_2 \le 1750 && \text{(Machine Hour Limit)} \\
& 0 \le x_1 \le 1000 && \text{(Bat Grip Supply)} \\
& 0 \le x_2 \le 1500 && \text{(Hockey Grip Supply)}
\end{aligned}
$$

Feasibility Space

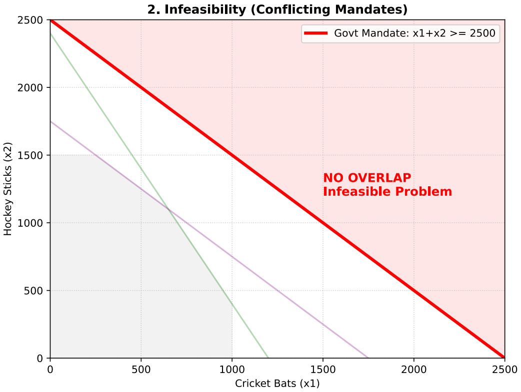


Key Observations:

- The shaded region is the **Feasible Set**.
- Any point inside is a possible production plan.
- The **Optimal Solution** occurs at a vertex (corner) where constraints intersect.

Issues #1: Infeasibility (No Solution)

What if the government mandates:
"Produce at least 2,500 total units to save jobs."
$x_1 + x_2 \geq 2500$
This line lies completely outside our resource capability (Wood/Machine).
There is no point that satisfies all conditions.



Issues #2: Unboundedness

What if we ignore raw material limits and only consider a lower bound?

"Produce at least 100 bats."

Without the wood/machine "ceiling," the profit can go to **infinity**.

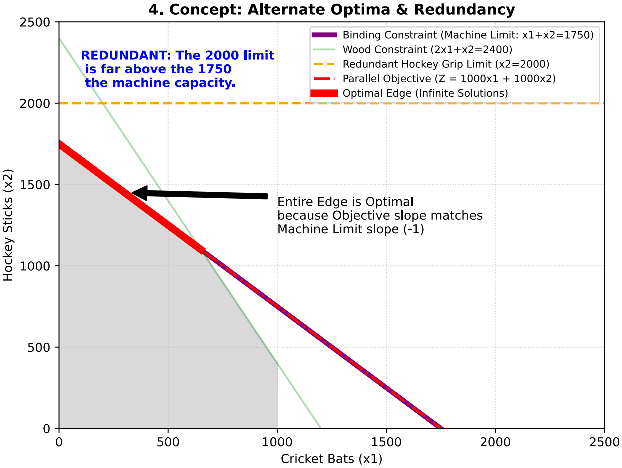
Grid Context: This usually means you forgot to model physical line limits!

Issues #3: Redundancy & Multiple Optima

Redundancy: *Hockey Grip Limit (2000)* is redundant. We run out of wood long before we can make 2000 sticks. Removing it doesn't change the answer.

Multiple Optima (Alternate Solutions):

- Occurs when the profit line is **parallel** to a constraint.
- You can shift production between x_1 and x_2 without changing the total profit.
- Excellent for Grid Operators to provide "flexibility."



Summary: Grid Perspective

Manufacturing Concept	**Grid Concept**
Wood / Machine Limits	Transmission Line Thermal Limits
Grip Stocks (Fixed supply)	Generator Pmax / Pmin
Maximizing Profit	Minimizing Social Cost / LMPs
Infeasibility	Load Shedding Required
Redundancy	Non-congested Transmission Lines

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Terms Paper Form is Due before Next Week's Class.