EET 110 Homework 2: Graphical Representation of LP

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Solved Problem

Maximize the objective function:

$$\begin{aligned} z &= 2x + 3y && \text{(Objective)} \\ -3x + y &\leq 2 && (L_1) \\ 4x + 2y &\leq 44 && (L_2) \\ 4x - y &\leq 20 && (L_3) \\ -x + 2y &\leq 14 && (L_4) \\ x, y &\geq 0 && \text{(Non-negativity)} \end{aligned}$$

Solution Procedure

1. Identification of Vertices

By solving the intersection of the boundary lines, we identify the vertices of the feasible region:

- **A**: $(0, 0)$ — Intersection of $x = 0, y = 0$.
- **B**: $(5, 0)$ — Intersection of $y = 0$ and $4x - y = 20$.
- **C**: $(7, 8)$ — Intersection of $4x - y = 20$ and $4x + 2y = 44$.
- **D**: $(6, 10)$ — Intersection of $4x + 2y = 44$ and $-x + 2y = 14$.
- **E**: $(2, 8)$ — Intersection of $-x + 2y = 14$ and $-3x + y = 2$.
- **F**: $(0, 2)$ — Intersection of $-3x + y = 2$ and $x = 0$.

2. Objective Function Evaluation

The gradient of the objective function is $\nabla z = (2, 3)$. We move the objective line $2x + 3y = k$ in the direction of this gradient to find the maximum value.

- At $D(6, 10)$: $z = 2(6) + 3(10) = 12 + 30 = \mathbf{42}$.

Draw Feasible Space of Following LPs

Problem 1

Solve the following Linear Programming problem graphically by sketching the constraint region and the optimal level set:

$$\begin{aligned} \text{minimize} \quad & x + y \\ \text{subject to} \quad & -x + y \leq 3 \\ & 2x + y \leq 18 \\ & y \geq 6 \\ & 0 \leq x, y \end{aligned}$$

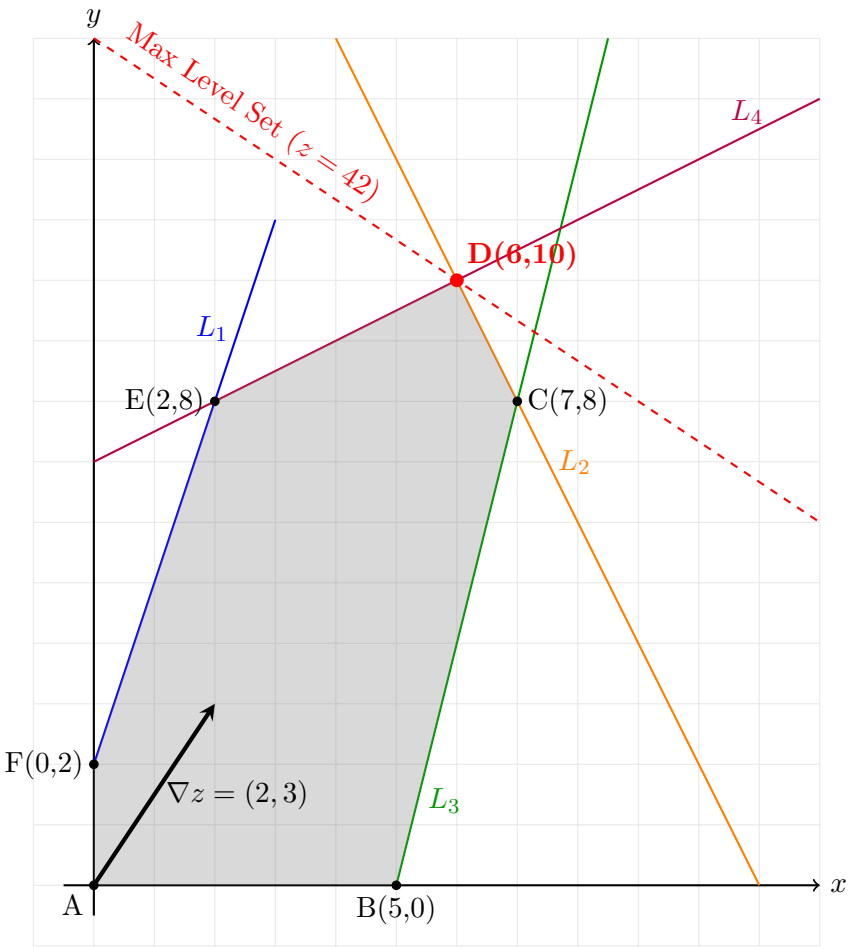

Figure 1: Feasible region and optimal solution for LP 1(a). The shaded region represents the feasible space. The dashed red line indicates the level set of the objective function reaching the optimal vertex $D(6, 10)$.

Problem 2

Solve the following Linear Programming problem graphically by sketching the constraint region and the optimal level set:

$$\begin{aligned} \text{maximize} \quad & 3x + 2y \\ \text{subject to} \quad & x + 4y \leq 16 \\ & x - 2y \leq -1 \\ & 1 \leq y \leq 4 \\ & 0 \leq x \leq 4 \end{aligned}$$

Problem 3: Parametric LP

Graph the following function of α by graphically solving the necessary Linear Programs:

$$v(\alpha) := \begin{bmatrix} \text{maximize} & x_1 + \alpha x_2 \\ \text{subject to} & x_1 - x_2 \leq 4 \\ & x_1 + x_2 \leq 6 \\ & -3x_1 + x_2 \leq -6 \\ & 0 \leq x_1, x_2 \end{bmatrix}$$