Indian Institute of Technology (IIT) Roorkee
Department of Electrical Engineering
**EET-110: Power & Energy Management-II**
**End-Term Examination (Spring 2025–26)**

**Your Set**

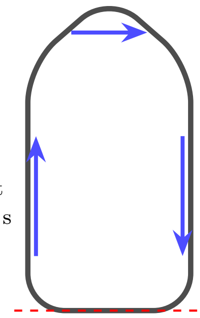
**Timing: 3 hrs** **Date: May 8, 2026** **Max. Marks: 50**

## Instructions

- Answer all questions. Each question must begin on a **NEW PAGE**.
- Provide complete justifications for answers. Mention assumptions & conditions wherever needed.

**[May The Eighth be With You]**

1. It is the 2026 season and **Ferrari** has pulled off the blockbuster signing of **Max Verstappen** (#33). You are the lead energy strategist on the pit wall. Under the new regulations, Ferrari's SF-26 power unit has two sources: an Internal Combustion Engine (ICE) and a battery. The power split is roughly 50/50. Consider a simplified model of **one lap** of a circuit divided into three segments as shown below.




Under the 2026 regulations, the car's power unit has two sources: a V6 ICE capped at 350 kW and a Battery that can either deploy up to 350 kW from the battery or harvest up to 200 kW back into it. The battery starts each lap with a state-of-charge of 3 MJ and must never drop below 0.5 MJ or exceed 4 MJ at any point during the lap; moreover, at least 2 MJ must remain at the end of the lap to sustain the next lap. To remain competitive, the car must deliver at least 550 kW of total power on Segment 1, at least 150 kW on Segment 2 (a slow corner where braking energy can be recovered), and at least 600 kW on Segment 3. The team's fuel strategist estimates that each segment burns fuel at a rate proportional to the ICE power output: specifically 0.09 g per kW·s on the two straights and 0.06 g per kW·s through the corner. The total fuel available for the lap is 800 g.

**(a)** The team wants to *maximize the total energy delivered over the lap* (a proxy for minimizing lap time). Write the complete LP formulation. **[4]**

**(b)** Suppose due to track evolution the minimum power requirement is uncertain: $P_k^{\min} = \bar{P}_k + \rho\, u_k$ where $\bar{P}_k$ are the nominal values above, $\rho = 40\,\text{kW}$ is the uncertainty radius, and the uncertainty vector $u = (u_1, u_2, u_3)$ satisfies $\|u\|_2 \le 1$. Re-formulate the power-requirement constraints so that they hold for *all* admissible $u$. What class of optimization problem does this become? **[4]**

**(c)** After solving the LP in part (a), the team observes that the dual variable associated with the fuel-budget constraint is $\lambda^\star = 0.6\,\text{kJ/g}$. The FIA proposes increasing the fuel allowance from 800 g to 820 g for the next race. How much additional energy (approximately) can the team deliver per lap? Would the team benefit from this change if extra fuel costs them Rs. 1.5 per gram and each extra kJ of delivered energy is worth Rs. 0.1 in time saved? **[2]**

2. Consider a power network modeled as a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of transmission lines. Each bus $i \in \mathcal{N}$ has a complex voltage $V_i = |V_i|\, e^{j\theta_i}$, a complex power injection $S_i^{\text{inj}} = P_i^{\text{inj}} + jQ_i^{\text{inj}}$, and possibly one or more generators. The network is described by the *admittance matrix* $Y \in \mathbb{C}^{n\times n}$, where $Y = G + jB$ with $G, B \in \mathbb{R}^{n\times n}$.

**(a)** Formulate a standard ACOPF problem for this network with (i) cost minimization objective and (ii) loss minimization objective. **[3+4]**

**(b)** Identify the mathematical source(s) of non-convexity in the above formulation and provide one convex relaxation of the same ACOPF problem (cost minimization only). **[1+4]**

**(c)** Let the optimal value of the original ACOPF be $\mathcal{O}^\star$ and the relaxed problem's optimal value be $\mathcal{O}^r$. Consider both $\mathcal{O}^\star \le \mathcal{O}^r$ and $\mathcal{O}^\star \ge \mathcal{O}^r$. Argue which can occur and when. **Answer should not exceed three text lines; mathematical justification beyond that is allowed.** **[1+1]**

3. Consider the optimization problem: 
$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|^2 \\ \text{s.t.} \quad & Cx = d \end{aligned}$$

Suppose $\hat{x}$ and $z$ satisfy
$$A^T A\,\hat{x} + C^T z = A^T b \text{ and } C\hat{x} = d.$$

Prove that $\hat{x}$ is an optimal solution. **[3]**

4. Given sorted data points $\{y_1, \dots, y_m\} \subset \mathbb{R}$ with $y_1 \le \cdots \le y_m$, consider

$$\text{minimize}_x \quad \left\| \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} - \begin{bmatrix} x \\ \vdots \\ x \end{bmatrix} \right\|_p .$$

State the optimal $\hat{x}$ and its statistical meaning for $p = 1,\ 2,$ and $\infty$. **[3]**

5. **(a)** Reformulate the following as a standard LP (min over $x \ge 0$ with linear constraints): **[1]**

$$\min_{x \ge 0} \left( \max_{i \in \{1,2,3\}} \{a_i x + b_i\} \; + \; |x - 5| \right).$$

**(b)** Define the *Chebyshev center* of a polyhedron $\mathcal{P} = \{y \in \mathbb{R}^n \mid a_i^T y \le b_i,\ i = 1, \dots, k\}$ geometrically, and write the explicit LP formulation to find it. **[1]**

6. You are **Bob the Builder** and is constructing a solar-panel mounting structure on a rooftop. You need two resources: steel beams (S) and aluminium brackets (A). You can purchase from 2 suppliers.

| | Steel (kg) | Aluminium (kg) | Cost per bundle (Rs.) |
|---|---|---|---|
| Supplier 1 (bundle) | 2 | 1 | 12 |
| Supplier 2 (bundle) | 1 | 2 | 9 |
| Minimum requirement | 10 | 11 | |

Let $x_1, x_2 \ge 0$ be the number of bundles purchased from Suppliers 1 and 2.

**(a)** Write the primal LP (minimize cost). **[2]**

**(b)** Write the dual LP. Identify the dual variables and give their economic interpretation. **[3]**

**(c)** The optimal primal solution is $x_1^\star = 3,\ x_2^\star = 4$ (you may verify feasibility). Using *complementary slackness*, determine the optimal dual solution $(\lambda_1^\star, \lambda_2^\star)$. **[3]**

**(d)** Bob can get an additional kg of aluminium at the market for Rs. 1.5/kg. Should he buy it? Justify using the shadow price. **[2]**

7. State whether each statement is **True** or **False** and provide a one- or two-line justification (no justification $\Rightarrow$ no marks).

**(a)** Every LP is also an SOCP. **[2]**

**(b)** If the primal LP is unbounded, then the dual LP must be infeasible. **[2]**

**(c)** The set $\{x \in \mathbb{R}^2 : x_1^2 - x_2^2 \le 0\}$ is a convex set. **[2]**

**(d)** In the $\ell_1$-regularized least squares (LASSO), the optimal solution tends to be *sparse*, whereas $\ell_\infty$-regularization tends to make components *equal* in magnitude. **[2]**

**[This is The End! Hope Sky Won't Fall When We Meet Next Time!]**